A Method of Producing Broad-Band Circular Polarization Employing an Anisotropic Dielectric*

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Summary—**A procedure is described whereby it is possible to design circular polarizers for both waveguides and in window form to be used over a broad band of frequencies. The difference in phase constants for two mutually orthogonal *E* fields while propagating in an anisotropic dielectric is combined with the effect due to guide wall spacing to obtain a reasonably constant differential phase constant for the two fields over a broad frequency band. By properly choosing the length of the anisotropic dielectric in the direction of propagation, and orienting this dielectric properly with respect to an incident linearily-polarized wave, the transmitted wave is circularly polarized over a correspondingly broad band of frequencies.**

Introduction

VARIOUS methods[1-4] have been proposed for obtaining circular polarization of an electromagnetic wave in a waveguide or in free space. Some of these methods are frequency sensitive while others are not. This paper describes a method for obtaining circular polarization over a broad band of frequencies by making use of an artificial anisotropic dielectric.

A wave in a waveguide may be said to be circularly polarized if the following conditions are met. The wave shall consist of two equal components in space quadrature (each being a dominant mode) traveling in the same direction. The electric vectors of the two components shall be 90° out of phase with each other. The equality of the two electric field vectors is necessitated by the fact that a circularly-polarized wave can be propagated in a waveguide without loss of circularity only if the two components that go to make it up are propagated with the same velocity. In other words, what is required is a guide that is doubly symmetric in cross section such as either a square or a circle.

The method described here makes use of a section containing an artificial anisotropic dielectric in the waveguide (which is not necessarily doubly symmetric), disposed in such a way as to produce circular polarization at the interface furthest from the source of energy. If this interface is followed by a perfectly square or round guide, the circularly-polarized wave will propagate unaltered.

The use of an anisotropic dielectric is further extended to produce a polarizing window which can control the polarization emanating from a linearly-polarized antenna.

Analysis

Consider a rectangular waveguide as in Fig. 1, which is capable of transmitting both the TE_{10} and TE_{01} modes. This waveguide is filled with an anisotropic dielectric having the property that the dielectric constant for the electric vector in the *y* direction is greater than the dielectric constant for the electric vector in the *x* direction. If it is assumed that only the TE_{10} and TE_{01} modes are transmitted, then the *x*-polarized wave will "see" only ϵ_x and the *y*-polarized wave will "see" only ϵ_x.

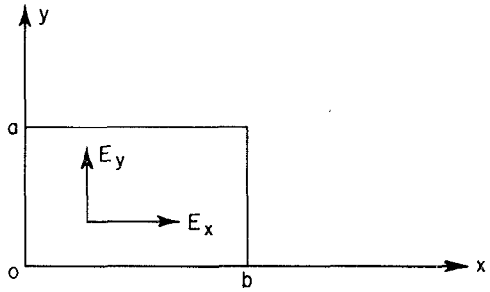

Fig. 1—Coordinate system and waveguide dimensions.

These two waves will behave as though the guide were filled with an isotropic dielectric having dielectric constants ϵ_x and ϵ_y, respectively. For these two polarizations, the propagation constants are, respectively,

$$\beta_x = \beta_0 \sqrt{k_x - \left(\frac{\lambda}{2a}\right)^2} \tag{1}$$

$$\beta_y = \beta_0 \sqrt{k_y - \left(\frac{\lambda}{2b}\right)^2} \tag{2}$$

where $\beta_0 = \omega\sqrt{\mu_0 \epsilon_0}$, k_x and k_y are the relative dielectric constants of the anisotropic medium in the directions indicated by the subscript, and λ is the free-space wavelength of the wave.

For the case where $\epsilon_y > \epsilon_x$ and $b > a$, the above relations plot as shown in Fig. 2. At some frequency ω_0 the slopes of the two curves are equal, and this frequency can be

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[1] "Preliminary Instruction Book for the M4902 Airborne 'S' Band Circularly-Polarized Radiator," Radio Res. Lab., Harvard University Rep. No. 411-1B-71; March 12, 1945.

[2] "Quarter-Wave Plate for Broad-Band Circular Polarization," M.I.T. Rad. Lab. Ser., Rep. No. 769, McGraw-Hill Book Co., Inc., New York, N. Y.; January 28, 1946.

[3] R. M. Brown and A. J. Simmons, "Dielectric Quarter-Wave and Half-Wave Plates in Circular Waveguide," Naval Res. Lab. Rep. No. 4218; November 10, 1953.

[4] A. J. Simmons, "A Method of Producing Broad-Band Circular Polarization in Square Waveguide," Naval Res. Lab. Rep. No. 4286; January 28, 1954.

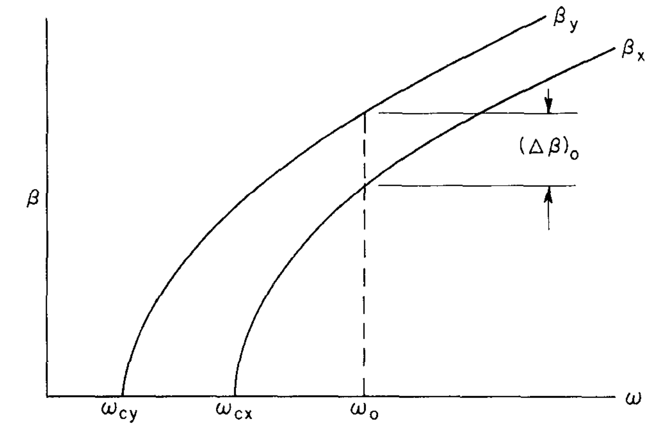


Fig. 2—Phase constant vs frequency.

taken as the center of a band of frequencies over which $(\beta_y - \beta_x)$ remains very nearly constant. Upon solving for the frequency at which the slopes of the two propagation functions are equal, there is obtained

$$\omega_0 = \frac{\pi}{\sqrt{\mu_0\epsilon_0(k_y - k_x)}} \sqrt{\frac{1}{a^2}\frac{k_y}{k_x} - \frac{1}{b^2}\frac{k_x}{k_y}}. \quad (3)$$

From (3) it follows that

$$\frac{4(k_y - k_x)}{\lambda^2} = \frac{1}{a^2}\frac{k_y}{k_x} - \frac{1}{b^2}\frac{k_x}{k_y} \quad (4)$$

where λ_0 is the free-space wavelength corresponding to ω_0. If λ_0 is fixed at some particular value, then a curve such as Fig. 3 can be drawn showing the relationship between a and b required to satisfy (4).

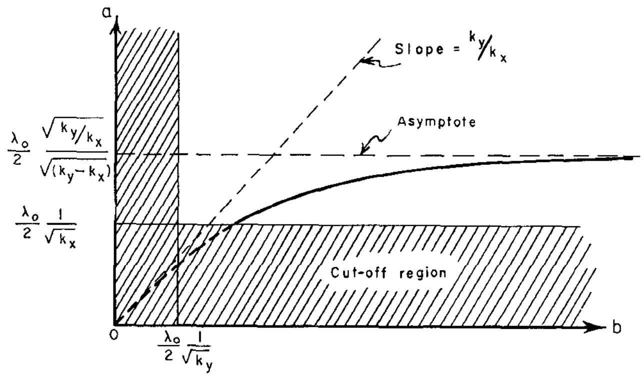


Fig. 3—Usable range of guide dimensions.

It should be noted that this curve is asymptotic to

$$\frac{\lambda_0}{2}\frac{\sqrt{k_y/k_x}}{\sqrt{k_y - k_x}}$$

as b becomes infinitely large. The case for b = infinity will be taken up in greater detail later when the subject of a polarizing window is discussed.

Eq. (4) is good provided neither a nor b becomes so small as to result in the guide's being cut off at the wavelength λ_0. Since a practical design requires $b > a$, and since $\epsilon_y > \epsilon_x$, the guide will cut off first for the x-polarized wave. The cutoff wavelength for the x-polarized wave is

$$\lambda_{cx} = 2a\sqrt{k_x}, \quad (5)$$

from which it can be seen that the dimension a cannot be less than $\lambda_0/2\sqrt{k_x}$. Hence in Fig. 3, only the solid portion of the a vs b curve can be used. From the point of view of the simplicity of the transition pieces from rectangular to round or square guide, it would be best to choose a square cross section for the quarter-wave plate. However, with the k_y and k_x available, such a choice would either result in a quarter-wave plate of large cross section or it would put ω_0 too close to the cutoff region. To permit ω_0 to be somewhere near the center of the band of frequencies to be considered, it is usually necessary to choose a point on the curve of Fig. 3 such that at the lowest frequency to be considered the quarter-wave plate is not cut off.

Having established the dimensions of the cross section, a curve of $\Delta\beta$ vs frequency can be computed for the frequency band under consideration. From the average value of $\Delta\beta$ over this band, a length can be chosen for the quarter-wave plate such that $(\Delta\beta)_{ave}$ times the length l equals $\pi/2$ radians.

Up to this point, no consideration has been given to the problem of internal reflections within the quarter-wave plate due to mismatches at the two interfaces. If it is assumed that the guide walls are continued unbroken on either side of the quarter-wave plate, as would probably be the case for a round or square guide, then the complex transmission coefficients through the quarter-wave plate for the x and y polarizations of the wave are

$$\dot{k}_{tx} = \frac{1}{\cos\beta_x l + j\frac{1}{2}\left(\frac{Z_{0x}}{Z_{1x}} + \frac{Z_{1x}}{Z_{0x}}\right)\sin\beta_x l} \quad (6)$$

$$\dot{k}_{ty} = \frac{1}{\cos\beta_y l + j\frac{1}{2}\left(\frac{Z_{0y}}{Z_{1y}} + \frac{Z_{1y}}{Z_{0y}}\right)\sin\beta_y l}, \quad (7)$$

where $\dot{k}_{tx}$ and $\dot{k}_{ty}$ are the complex transmission coefficients for the x- and y-polarized waves. The quantities Z_{0x} and Z_{0y} are the guide impedances for the air-filled guide and Z_{1x} and Z_{1y} are the guide impedances in the region filled with the anisotropic dielectric. From (6) and (7) the actual differential phase shift between the incident waves and emergent waves will be

$$\Delta\phi = (\phi_y - \phi_x) = \arctan\left\{\frac{1}{2}\left[\frac{Z_{1y}}{Z_{0y}} + \frac{Z_{0y}}{Z_{1y}}\right]\tan\beta_y l\right\} - \arctan\left\{\frac{1}{2}\left[\frac{Z_{1x}}{Z_{0x}} + \frac{Z_{0x}}{Z_{1x}}\right]\tan\beta_x l\right\}. \quad (8)$$

The ratio of the absolute values of $\dot{k}_{tx}$ and $\dot{k}_{ty}$ when averaged over the frequency band under consideration will serve to determine the relative magnitudes of the incident x- and y-polarized waves necessary to produce circular polarization in the emerging wave. The incident waves must meet the following requirement in order to produce circular polarization, namely,

$$\left|\frac{E_{ix}}{E_{iy}}\right| = \left|\frac{\dot{k}_{ty}}{\dot{k}_{tx}}\right|_{\text{ave}} \tag{9}$$

where the average is taken over the frequency band under consideration.

This requirement follows from the fact that the transmitted waves must be equal in magnitude in order to have circular polarization.

This same anisotropic material when placed in a round guide leads to a problem, the exact solution of which is intractable. However, as shown later under the experimental work, such a round guide behaves qualitatively in the same manner as a square guide having the same cutoff wavelength in the dominant mode.

The Polarizing Window

As stated above, when b is allowed to approach infinity the dimension a approaches

$$\frac{\lambda_0}{2} \frac{\sqrt{k_y/k_x}}{\sqrt{k_y - k_x}},$$

and for the y-polarized wave, the guide would become a parallel-plate transmission line. For the x-polarized wave, the guide would be infinitely deep. If a number of these guides were stacked above each other, there would result a configuration as shown in Fig. 4. This could be made to serve as a polarizing window when placed in front of a linearly-polarized antenna. In Fig. 2, the cutoff frequency for the y-polarized wave would become zero and the phase constant through the material would become a straight line function of frequency. As in the case of the quarter-wave plate in the waveguide, (6), (7), and (8) apply with Z_{0x} and Z_{0y} equal to the impedance of free space. The impedances Z_{1x} and Z_{1y} are, respectively,

$$Z_{1x} = \frac{\sqrt{\mu_0/\epsilon_0}}{\sqrt{k_x - \left(\frac{\lambda}{2a}\right)^2}} \tag{10}$$

$$Z_{1y} = \frac{1}{\sqrt{k_y}} \sqrt{\frac{\mu_0}{\epsilon_0}}. \tag{11}$$

The Anisotropic Dielectric

The anisotropic dielectric used in the quarter-wave plates described in this paper is an artificial one composed of alternate sheets of polystyrene and air (or polyfoam). The sheets of polystyrene and air are of equal

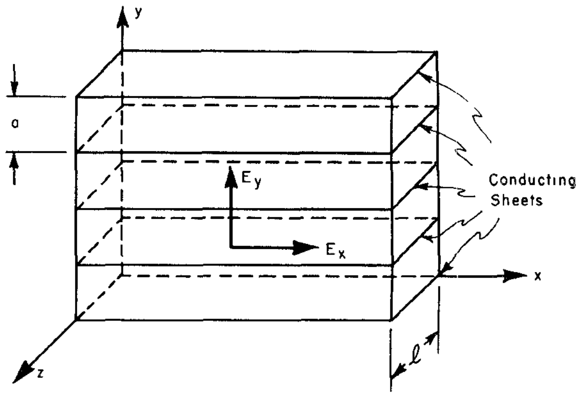


Fig. 4—Construction details of polarizing window.

thickness as shown in Fig. 5. If the wavelength used in the quarter-wave plates is very much greater than the thickness of the sheets of the anisotropic dielectric, the dielectric constants of the anisotropic dielectric can be computed on the basis of a dc field. Upon doing this it is found that

$$\epsilon_x = \frac{2\epsilon_p \epsilon_f}{\epsilon_p + \epsilon_f} \tag{12}$$

$$\epsilon_y = \frac{\epsilon_p + \epsilon_f}{2}, \tag{13}$$

where ϵ_x and ϵ_y are the dielectric constants for x- and -polarized fields, respectively, and ϵ_p and ϵ_f are the dielectric constants of polystyrene and air (or polyfoam) respectively. These equations are obviously applicable to relative dielectric constants also. Tests on the quarter-wave window placed in front of a horn antenna up to frequencies where the thickness of the polystyrene was about $\frac{1}{3}$ of a wavelength indicate that the use of these dc dielectric constants is justifiable.

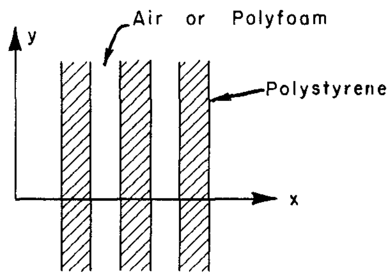


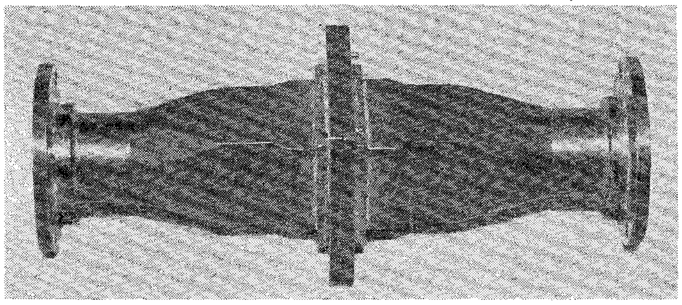
Fig. 5—Artificial anisotropic dielectric.

For a polystyrene and air anisotropic dielectric, the ϵ_x and ϵ_y are 1.438 ϵ_0 and 1.780 ϵ_0, respectively. If polyfoam is used instead of air, they are 1.470 ϵ_0 and 1.795 ϵ_0, respectively.

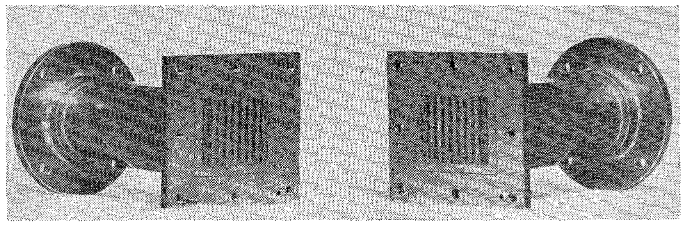
Experimental Results

Quarter-Wave Plates

A number of quarter-wave plates embodying the principles stated above were built and tested. The first of these, shown in the photographs of Fig. 6, had the di-



(a)



(b)

Fig. 6—Photographs of quarter-wave plate.

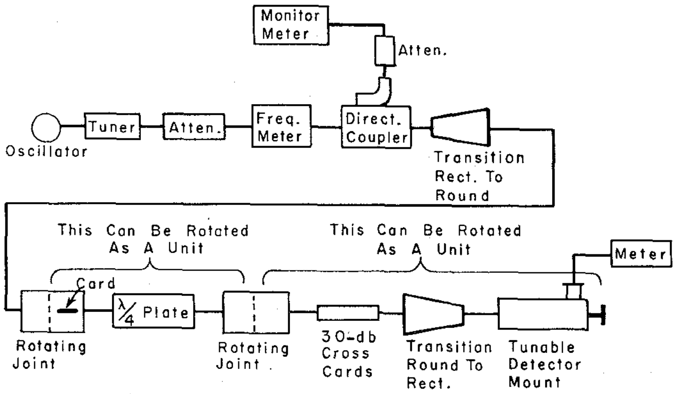


Fig. 7—Test equipment used to measure axial ratio of quarter-wave plates in a guide.

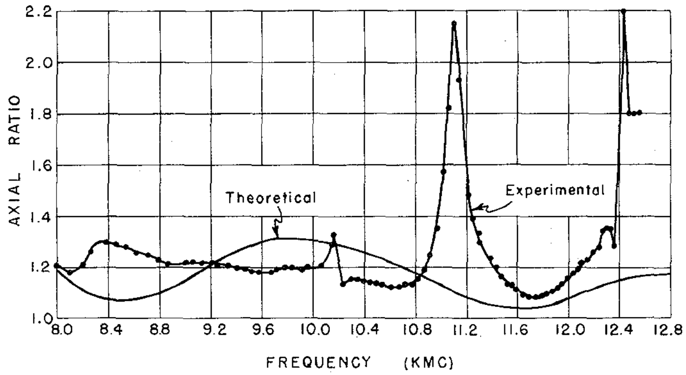


Fig. 8—Axial ratio vs frequency for λ/4 plate in 2.8×2.3-cm guide.

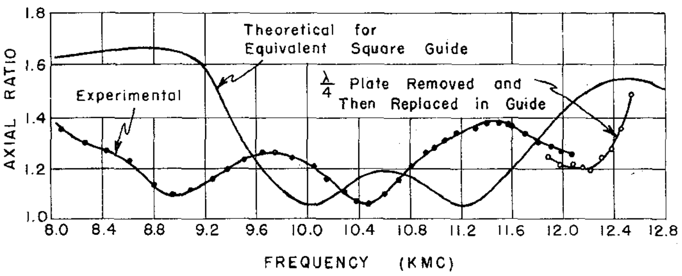


Fig. 9—Axial ratio vs frequency for λ/4 plate in 15/16-inch circular guide.

mensions listed in Table I, below. This quarter-wave plate was designed for a rectangular guide so that advantage could be taken of the differential phase shift inherent in a rectangular guide regardless of the presence of an anisotropic medium.

TABLE I

Dimension a	$=2.30$ cm
Dimension b	$=2.80$ cm
Length l	$=3.30$ cm
Dielectric thickness	$=0.159$ cm
Air thickness	$=0.159$ cm
ϵ_y	$=1.780\ \epsilon_o$
ϵ_x	$=1.438\ \epsilon_o$

This quarter-wave plate was tested for axial ratio vs frequency using the circuit of Fig. 7. The results obtained are shown in Fig. 8, the anomalous behavior at 11.1 and 12.45 kmc being caused by resonance within the transition pieces. The theoretical performance is drawn on the figure as a comparison.

In order to eliminate the transition pieces, a quarter-wave plate, employing the same artificial anisotropic dielectric, was built into a round guide of 15/16-inch nominal inside diameter. This plate was 4.83 cm long, this length being based on the calculated performance of a hypothetical square guide having the same cutoff wavelength as the actual round guide used. In actuality an error in calculation resulted in the length of 4.83 cm being about 10 per cent longer than it should have been. The axial ratio vs frequency obtained for this quarter-wave plate is shown in Fig. 9. In this case, excellent results were obtained over a 1.56 to 1 frequency band. The theoretical performance of the hypothetical square guide quarter-wave plate of length 4.37 cm is shown for comparison. These measurements show that less differential phase shift per cm of dielectric is obtainable in the round guide than in the square guide, and that the mismatch at the interfaces does not seem to be as poor as in the case of the square guide. This may be explained qualitatively as follows. Consider the case of both guides transmitting waves polarized along the axis of greatest dielectric constant (the y axis in these reports). In the case of the square guide, there are no field components in the x direction and so the wave "sees" only ϵ_y. In the case of the round guide, off the diameter, there are components of electric field in both the x and y directions, the field in the y direction predominating. In any event, for the purposes of determining the propagation constant for this wave, the wave "sees" a dielectric constant somewhat less than ϵ_y. In a similar manner an x-oriented wave propagating in the circular guide will "see" a dielectric constant somewhat greater than ϵ_x. Hence the differential phase shift between x- and y- oriented waves will not be as great as it would in the perfectly square guide. On the other hand, this effect should improve the variation of the ratio of transmission coefficients, and result in a better axial ratio over the frequency band than would be obtainable in square guide.

Polarizing Windows

In Fig. 10 is shown a photograph of a polarizing window built in accordance with the above theory. The dimensions of the window are given in Table II. Measurements were made on this window by illuminating it with a 6×6 inch horn in which were placed several 700-ohm absorbing cards perpendicular to the incident polarization. This was done to absorb any cross-polarized reflections from the window. The incident polarization is inclined at an angle of 42.5° with respect to the metal strips of the window. This angle was determined by test and deviates from 45° because the average transmission coefficients for the x- and y- polarized waves are not equal to each other over the frequency band under consideration. The beam from the window was directed into a 7-foot cube "dark room" in which was placed a polarization-sensitive detector. Measurements were made, directly on the beam axis, of axial ratio over a band of frequencies extending from 8.2 kmc to 17.2 kmc. The results plotted in Fig. 11 show that the axial ratio does not exceed 1.5 over a 2.1 to 1 frequency band. It further appears that the useful bandwidth of the window has not been realized.

Conclusion

By means of the foregoing design procedure, it is possible to build circular polarizers for both waveguides and in the form of windows which will operate over a very broad band of frequencies. The design procedure did not take into account either impedance matching or "end effects" in the case of the window. If both of these were taken into account either analytically or experimentally, it should be possible to secure even better results than those obtained here.

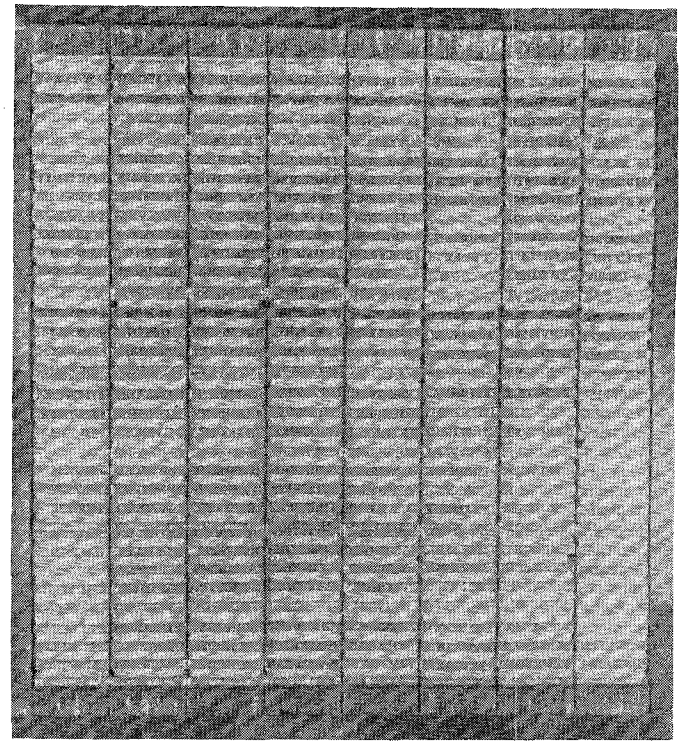


Fig. 10—Photograph of polarizing window.

TABLE II

Dimensions of Polarizing Window

$a=2.64$ cm	$\epsilon_y=1.79$	$\lambda_o=2.71$ cm
$l=2.60$ cm	$\epsilon_x=1.47$	$f_o=11.$ kmc
		$(\Delta\beta)_o\, l=82.5°$

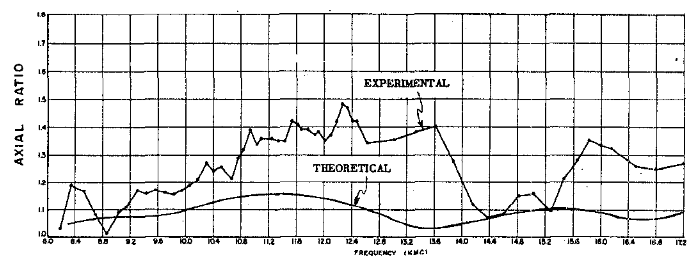


Fig. 11—Axial ratio vs frequency for polarizing window.